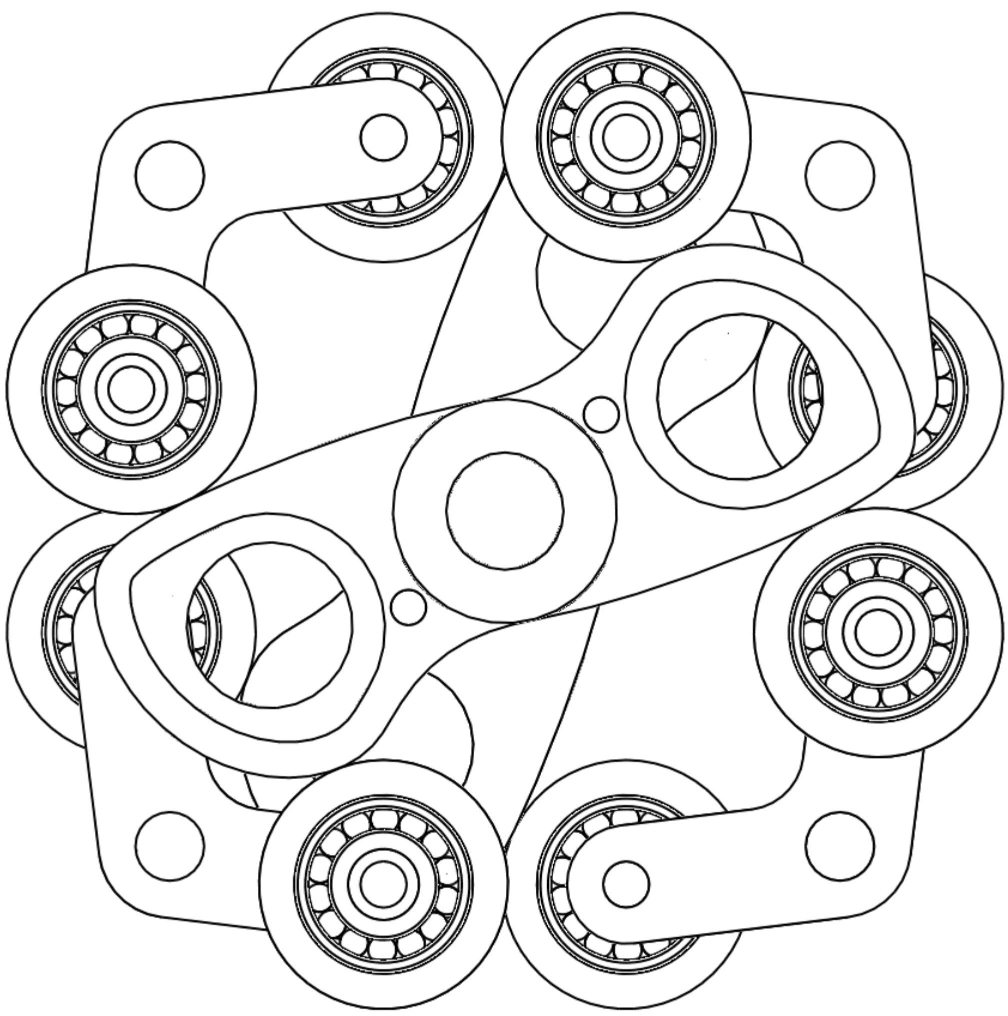
Mathematical solution of the Marchetti cams



By Wesley Peijnenburg

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Step by step derivation

The analytical solution of the Marchetti cams is derived in steps. The method is based on deriving the solution for a stationary cam, meaning that the rocker assembly is assumed to move around the main axis while the central cams are stationary.

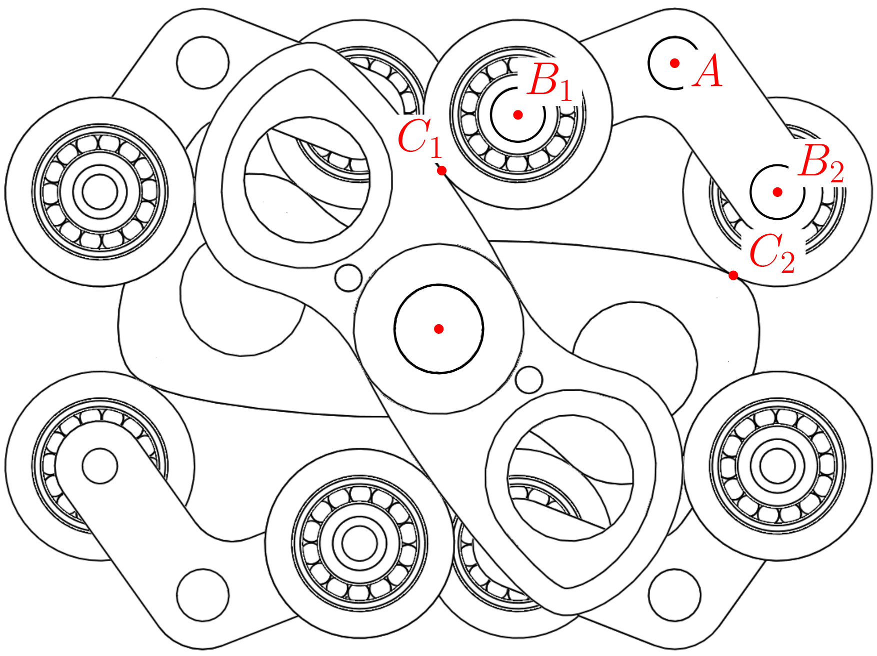

Figure 1: Sketch of the marchetti engine and parameters

Firstly, variables are defined. Figure 1 shows the pivoting point of the rocker arms (Point A), wheel centrepoints (B_1, B_2) and contact point between wheel and cam (C_1, C_2). The set of parameters required for the model are defined as follows:

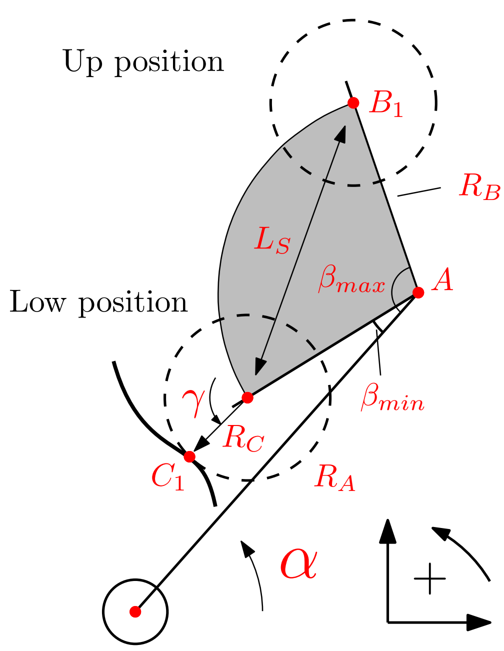

Figure 2: Parameter definition

Parameter	Value	Description
A	-	Rocker pivoting point
B	-	Wheel centre point
C	-	Contact point wheel and cam
R_A	170mm	Radius center to pivoting point
R_B	85mm	Rocker arm length
R_C	47mm	Wheel radius
L_S	-	Cylinder stroke
α	-	Angle to point A; main axis orientation
β_{max}	100°	Maximum angle of rocker arm
β_{min}	20°	Minimum angle of rocker arm
$\Delta\beta$	-	Difference β_{max} and β_{min}
γ	-	Angle from B_1 to contact point C_1

The solution starts with point A, from which point B is obtained and finally contact point C.

Calculating point A

Point A moves around the origin in a circle with radius R_A. The X and Y coordinates of point A as function of angle α are calculated as follows:

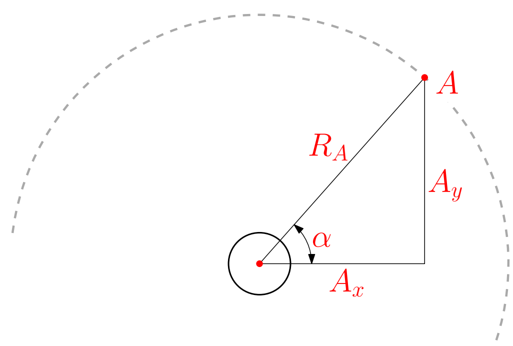

Figure 3: Point A

$$A_x = R_A \cos(\alpha)$$
$$A_y = R_A \sin(\alpha)$$

Calculating point B_1 and B_2

As viewed from point A, the arms are swinging from the low position β_{min} to the up position β_{max} and back. This movement is assumed to be sinusoidal and occurs twice per revolution.

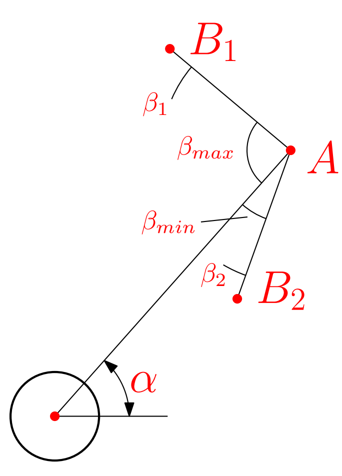

Figure 4: Point B

The two rocker arms are assumed to be a single rigid body, so the angle between the arms is constant. The angles β_1 and β_2 are defined for both arms as by the following functions:

$$\beta_1 = \beta_{min} + \frac{1}{2}\Delta\beta\ (1 - \cos(2\alpha))$$
$$\beta_2 = \beta_{min} + \frac{1}{2}\Delta\beta\ (1 + \cos(2\alpha))$$

Where $\Delta\beta = \beta_{max} - \beta_{min}$ and the angle is defined relative to the line connecting point A to the origin (and not the coordinate system). At $\alpha = 0$, the arms are initiated at $\beta_1 = \beta_{min}$ and $\beta_2 = \beta_{max}$. This is shown in Figure 5 below.

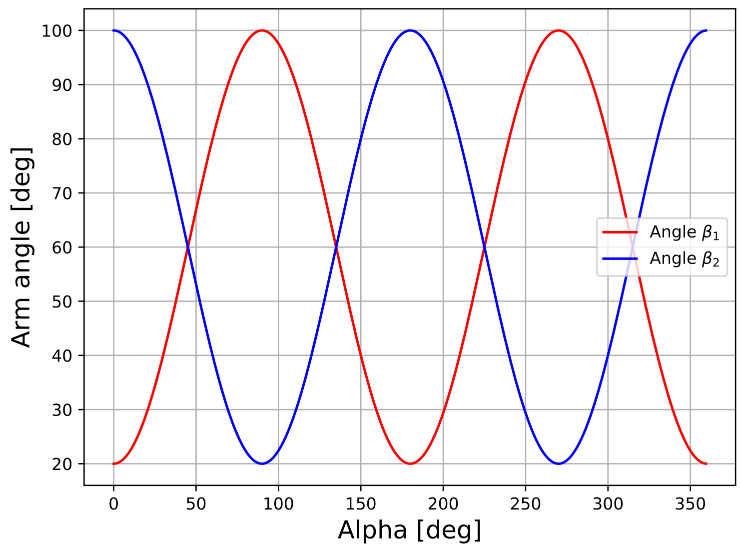

Figure 5: Angles β_1 and β_2 during a single engine revolution

With angles β_1 and β_2, the points B_1 and B_2 are derived. For all rotations, the positive direction is counter clockwise as shown in Figure 2.

$$B_{1x} = A_x + R_B \cos(\alpha + \pi - \beta_1)$$
$$B_{1y} = A_y + R_B \sin(\alpha + \pi - \beta_1)$$
$$B_{2x} = A_x + R_B \cos(\alpha + \pi + \beta_2)$$
$$B_{2y} = A_y + R_B \sin(\alpha + \pi + \beta_2)$$

Figure 6 below shows the trajectory of points B_1 and B_2 as they pivot around point A which is rotating around the centerpoint:

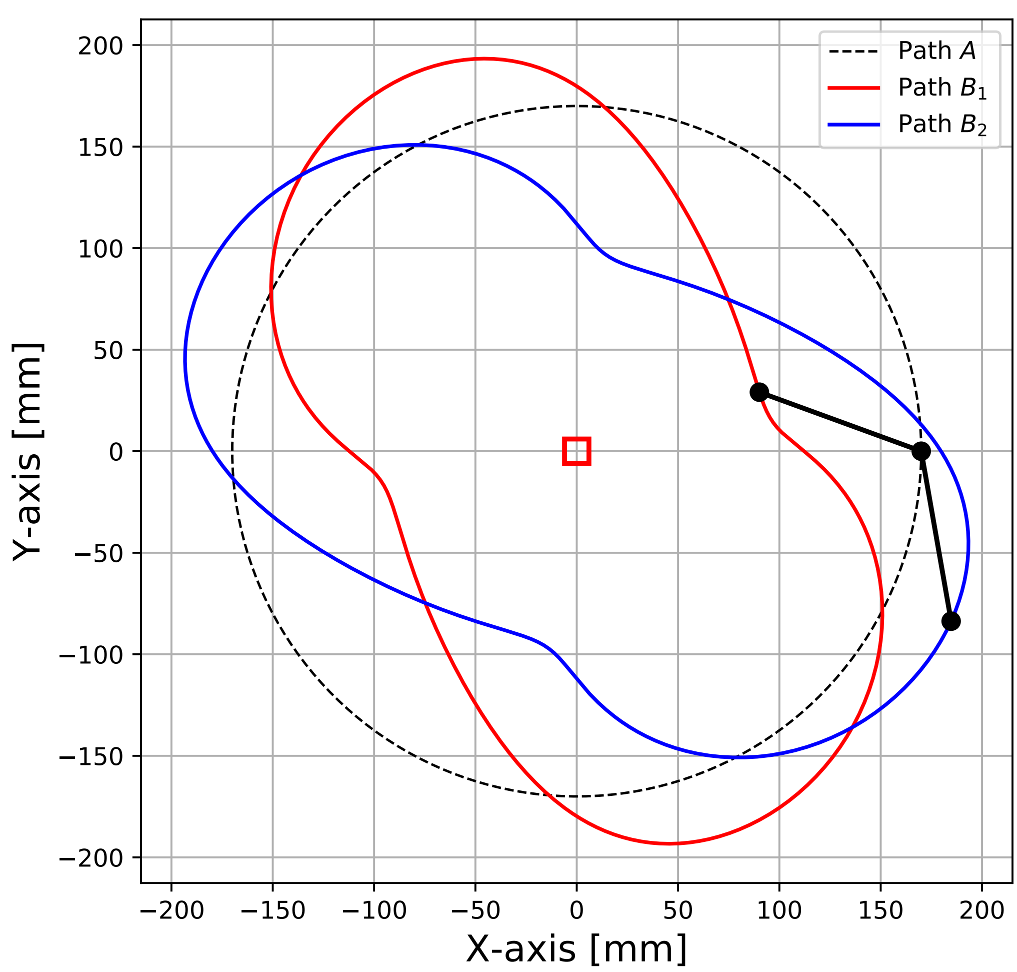

Figure 6: Paths B_1 and B_2 for a single engine revolution

Calculating point C_1 and C_2

As viewed from each point B, the path of point C runs parallel. So in order to determine point C, the slope of the curve (derivative with respect to α) needs to be known. This slope angle $\frac{dB}{d\alpha}$ is rotated 90 degrees inward, yielding angle γ. The angle γ points towards contact point C and is used to obtain point C at the distance of Rc from point B.

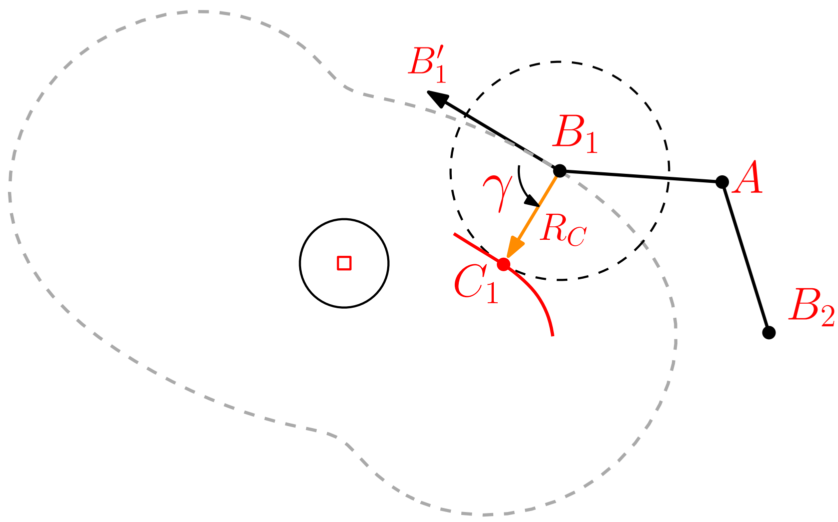

Figure 7: Point C

The slope of curve B_1 is equal to its derivative B_1'. This derivative is calculated in both x and y direction separately for both point B_1 and B_2:

$$\begin{aligned} B_{1x}' &= \frac{d}{d\alpha} B_{1x} \\ &= \frac{d}{d\alpha}\Big[R_A \cos(\alpha) + R_B \cos(\alpha + \pi - \beta_1)\Big] \\ &= -R_A \sin(\alpha) - R_B \sin\Big(\alpha + \pi - \beta_1\Big)\Big(1 - \Delta\beta \sin(2\alpha)\Big) \end{aligned}$$

$$\begin{aligned} B_{1y}' &= \frac{d}{d\alpha} B_{1y} \\ &= \ R_A \cos(\alpha) + R_B \cos\Big(\alpha + \pi - \beta_1\Big)\Big(1 - \Delta\beta \sin(2\alpha)\Big) \end{aligned}$$

$$B_{2x}' = -R_A \sin(\alpha) - R_B \sin\Big(\alpha + \pi + \beta_2\Big)\Big(1 - \Delta\beta \sin(2\alpha)\Big)$$

$$B_{2y}' = \ R_A \cos(\alpha) + R_B \cos\Big(\alpha + \pi + \beta_2\Big)\Big(1 - \Delta\beta \sin(2\alpha)\Big)$$

With the now known slope for curves B_1 and B_2, the slope is rotated 90° by simply switching the X and Y-axis: so $X_\gamma = -B_{1y}'$ and $Y_\gamma = B_{1x}'$. The vector $\frac{d}{d\alpha}B_1$ is then divided by it's own length to obtain the unit vector with slope angle γ. Point C is then obtained by multiplying with wheel radius R_C. Point C is then defined as follows:

$$C_{1x} = B_{1x} - R_C \frac{B_{1y}'}{\sqrt{B_{1x}'^2 + B_{1y}'^2}}$$

$$C_{1y} = B_{1y} + R_C \frac{B_{1x}'}{\sqrt{B_{1x}'^2 + B_{1y}'^2}}$$

$$C_{2x} = B_{2x} - R_C \frac{B_{2y}'}{\sqrt{B_{2x}'^2 + B_{2y}'^2}}$$

$$C_{2y} = B_{2y} + R_C \frac{B_{2x}'}{\sqrt{B_{2x}'^2 + B_{2y}'^2}}$$

The above equations are plotted in Figure 8 below:

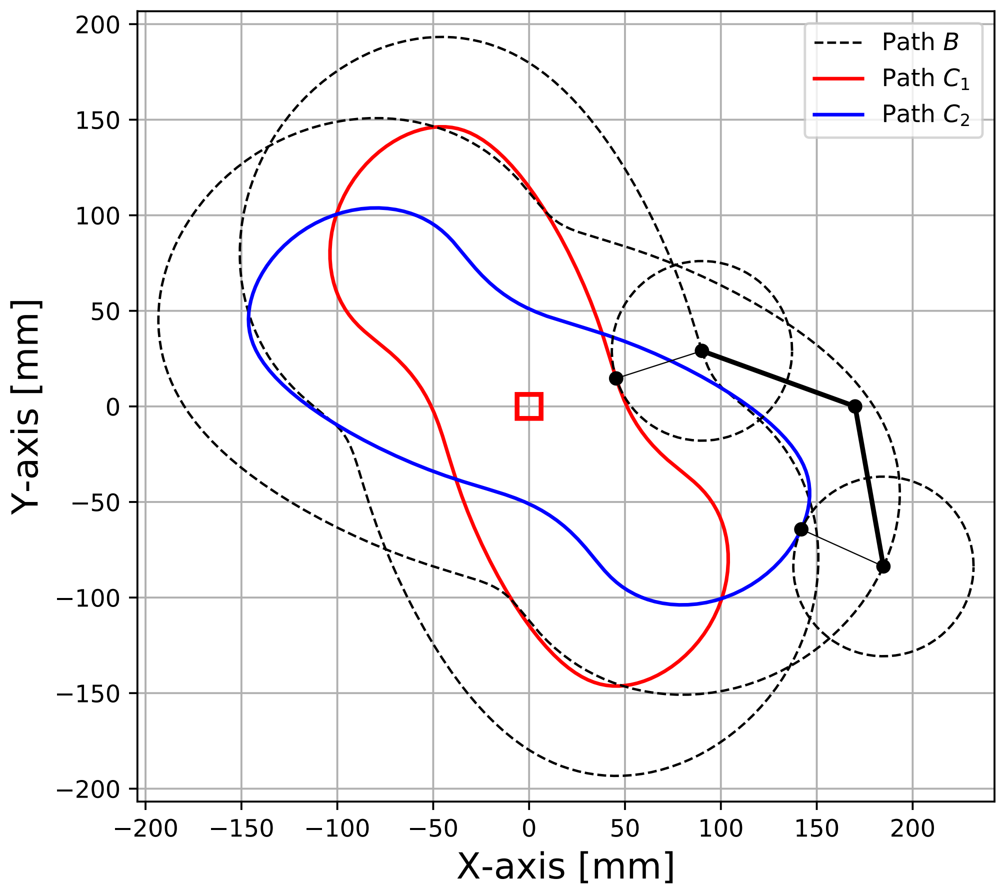

Figure 8: Paths C_1 and C_2 with wheels at $\alpha = 0$, the geometry of the cams

The piston stroke distance

The piston stroke L_S is determined by the arm length R_B and the angle $\Delta\beta$ as shown in Figure 2. It is obtained by considering a triangle from point A to the midpoint of L_S to point B:

$$L_S = 2\,R_B\,\sin\left(\frac{\Delta\beta}{2}\right)$$

Summary

The curves for the cams in the Marchetti engine are obtained for arbitrary input parameters. After altering the parameters, it is necessary to verify that the solution is physically possible. The first thing to check for is sharp edges or loops. Also the rocker angle β_{min} (and consequentially the cam contact angle) should not be chosen too close to zero, as this might cause excessive forces directed towards the pivoting point.

The website marchetti-engine.com also features an estimation of the parameters used in the original 1927 engine. It can be found under the section "*The math & how it's used to estimate the historical dimensions*".